Name:

MGF 1107 Section 0057
Quiz 4

Please show all of your work in a NEAT and ORGANIZED fashion.

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1. (4 points) Prove that the Euclidean metric is indeed a metric, where the Euclidean metric d measures the distance between (x_1, y_1) and (x_2, y_2) as follows:

$$d((x_1, y_1), (x_2, y_2)) = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}.$$

2. (3 points) Each of the following functions fails **exactly one** of the required properties of metrics (and therefore are not metrics). Identify the property each function fails, and explain your answer.

(a) $d((x_1, y_1), (x_2, y_2)) = 0$ for <u>any</u> points (x_1, y_1) and (x_2, y_2)

(b) $d((x_1, y_1), (x_2, y_2)) = \begin{cases} 0 \ \ if & (x_1, y_1) = (x_2, y_2) \\ 1 \ \ if & x_1 - x_2 > 0 \\ 2 \ \ if & x_1 - x_2 < 0 \end{cases}$

(c) $d((x_1, y_1), (x_2, y_2)) = \begin{cases} 0 \ \ if & (x_1, y_1) = (x_2, y_2) \\ -1 \ \ if & (x_1, y_1) \neq (x_2, y_2) \end{cases}$

3. (4 points) Prove the following proposition. Carefully justify each step of your proof.

Suppose l and m are two parallel lines. Prove that there exists a point which does not lie on either of the lines.

Hint: Begin the proof by constructing a line that intersects both l and m.

4. (4 points) Prove the following proposition. Carefully justify each step of your proof.

Suppose the ray $\overrightarrow{AD}$ is between rays $\overrightarrow{AB}$ and $\overrightarrow{AC}$, and the ray $\overrightarrow{AE}$ is between rays $\overrightarrow{AB}$ and $\overrightarrow{AD}$. Show that $\mu(\angle BAC) = \mu(\angle DAC) + \mu(\angle EAD) + \mu(\angle BAE)$.

Hint: Draw a picture!