MATH 146 Test 2 Study Guide

Indeterminate Forms

- L'Hôpital's Rule: Applies to the indeterminate forms $\frac{0}{0}$ and $\frac{\infty}{\infty}$
- Products: $0 \cdot \infty$
  - Rewrite the product as a quotient
- Differences: $\infty - \infty$
  - Rewrite the difference as a single fraction
- Powers: $0^0, \infty^0, 1^\infty$
  - Take ln of the limit; remember to exponentiate at the end

Improper Integrals

- "Horizontal infinity" (take the limit as t approaches the "problem point")
  1. $\int_a^\infty f(x)dx = \lim_{t\to\infty} \int_a^t f(x)dx$
  2. $\int_{-\infty}^b f(x)dx = \lim_{t\to-\infty} \int_t^b f(x)dx$
  3. $\int_{-\infty}^\infty f(x)dx = \int_{-\infty}^a f(x)dx + \int_a^\infty f(x)dx$; evaluate each integral on the right hand side as in (1) and (2)
- "Vertical infinity" (take the limit as t approaches the "problem point")
  1. Discontinuous at a: $\int_a^b f(x)dx = \lim_{t\to a^+} \int_t^b f(x)dx$
  2. Discontinuous at b: $\int_a^b f(x)dx = \lim_{t\to b^-} \int_a^t f(x)dx$
  3. Discontinuous at $a < c < b$: $\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$; evaluate each integral on the right hand side as in (1) and (2)

Sequences

- An infinite list of numbers: $\{a_n\}_{n=1}^\infty = a_1, a_2, a_3, \ldots$
- Convergence (finding $\lim_{n\to\infty} a_n$)
  - Divide top and bottom by power of n
  - Use related function and L'Hôpital's
  - Cancel factorials
- Monotonicity and Boundedness
  - Use derivative of the related function
  - The bounds you choose do not have to be the "best" bounds, just "safe" bounds
- Geometric sequence: $\{ar^n\}_{n=0}^\infty = a, ar, ar^2, \ldots$
  - Converges to 0 if $-1 < r < 1$
  - Converges to a if $r = 1$
  - Diverges otherwise

Series

- An infinite sum of numbers: $\sum_{n=1}^{\infty} a_n = a_1 + a_2 + a_3 + \cdots$
- Convergence is determined by the limit of the partial sums s_n
- **Geometric series:** $\sum_{n=0}^{\infty} ar^n$ converges only if $-1 < r < 1$, in which case $\sum_{n=0}^{\infty} ar^n = \frac{1st\ term}{1-ratio}$
- **p-series:** $\sum_{n=1}^{\infty} \frac{1}{n^p}$ converges if $p > 1$, diverges if $p \leq 1$
- **Telescoping series:** Partial sums s_n collapse; take limit of partial sums to determine convergence

Series Tests

- **Test for Divergence:** If $\lim_{n\to\infty} a_n$ DNE or $\lim_{n\to\infty} a_n \neq 0$, then the series $\sum_{n=1}^{\infty} a_n$ diverges.
- **Integral Test:** Suppose f is continuous, positive, and decreasing, and let $a_n = f(n)$. Then
    - If $\int_1^{\infty} f(x)dx$ converges, then $\sum_{n=1}^{\infty} a_n$ converges.
    - If $\int_1^{\infty} f(x)dx$ diverges, then $\sum_{n=1}^{\infty} a_n$ diverges.
- **Comparison Test:** Suppose that $\sum a_n$ and $\sum b_n$ are series with positive terms.
    - If $\sum b_n$ converges and $a_n \leq b_n$ for all n, then $\sum a_n$ also converges.
    - If $\sum b_n$ diverges and $a_n \geq b_n$ for all n, then $\sum a_n$ also diverges.
- **Limit Comparison Test:** Suppose that $\sum a_n$ and $\sum b_n$ are series with positive terms. If $\lim_{n\to\infty} \frac{a_n}{b_n} = c$, where c is finite and $c \neq 0$, then either both series converge or both series diverge.
- **Alternating Series Test:** If the alternating series $\sum_{n=1}^{\infty} (-1)^{n-1} b_n \quad (b_n > 0)$ satisfies $b_{n+1} \leq b_n$ and $\lim_{n\to\infty} b_n = 0$, then the series converges.
    - If $\sum_{n=1}^{\infty} (-1)^{n-1} b_n$ converges, then the following gives an estimate of error for the nth partial sum s_n: $|E_n| \leq b_{n+1}$.
- **Absolute Convergence:** A series $\sum a_n$ converges absolutely if the series of absolute values $\sum |a_n|$ converges.
    - A series $\sum a_n$ converges conditionally if it converges, but does not converge absolutely.
- **Ratio Test:** Let $L = \lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right|$.
    - If $L < 1$, then the series $\sum_{n=1}^{\infty} a_n$ converges (absolutely).
    - If $L > 1$, then the series $\sum_{n=1}^{\infty} a_n$ diverges.
    - If $L = 1$, no conclusion can be drawn.
- **Root Test:** Let $L = \lim_{n\to\infty} \sqrt[n]{|a_n|}$.
    - If $L < 1$, then the series $\sum_{n=1}^{\infty} a_n$ converges (absolutely).
    - If $L > 1$, then the series $\sum_{n=1}^{\infty} a_n$ diverges.
    - If $L = 1$, no conclusion can be drawn.