MATH 125 Fall 2019
Test 2 Practice Problems
(This is NOT a comprehensive review!!!)

1. Find an equation of the tangent line to the curve at the given point.

$$y = 4x^3 - \sqrt{x} + 5, \qquad (1, 8)$$

x_1, y_1

same notation $\left\{ \begin{array}{l} \frac{d}{dx}(\sqrt{x}) = \frac{1}{2\sqrt{x}} \\ (\sqrt{x})' = \frac{1}{2\sqrt{x}} \end{array} \right.$

$$y' = 12x^2 - \frac{1}{2\sqrt{x}}$$

Plug in $x = 1 \rightarrow$ $y' = 12 - \frac{1}{2}$

$$y' = \frac{24 - 1}{2} = \underbrace{\frac{23}{2}}_{m}$$

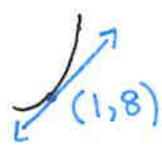
$\Longrightarrow$ equation of tangent line:

$$y - y_1 = m(x - x_1)$$

$$y - 8 = \frac{23}{2}(x - 1)$$

$$\boxed{y = \frac{23}{2}(x - 1) + 8}$$

$\hookrightarrow$ positive slope



2. Differentiate the function.

   a. $f(x) = x^{1/5} \tan x$

Product Rule: 1st times derivative of 2nd plus 2nd times derivative of 1st

$$f'(x) = x^{1/5} \frac{d}{dx}(\tan x) + (\tan x) \frac{d}{dx}(x^{1/5})$$

$$f'(x) = x^{1/5}(\sec^2 x) + (\tan x)\left(\frac{1}{5} x^{\frac{1}{5} - 1}\right)$$

$$f'(x) = \boxed{x^{1/5}(\sec^2 x) + (\tan x)\left(\frac{1}{5} x^{-4/5}\right)}$$

b. $g(x) = \frac{x^2+4x-1}{x^2+9}$

$$g'(x) = \frac{(x^2+9)(x^2+4x-1)' - (x^2+4x-1)(x^2+9)'}{(x^2+9)^2}$$

$$= \boxed{\frac{(x^2+9)(2x+4) - (x^2+4x-1)(2x)}{(x^2+9)^2}}$$

Quotient Rule: bottom times derivative of top, minus top times derivative of bottom, all over bottom^2.

c. $h(x) = \underbrace{\sqrt[3]{\vphantom{x}}}_{1}\underbrace{5x+6+\frac{1}{x}}_{2}$

Chain Rule:

$F(x) = f(g(x))$

$F'(x) = f'(g(x)) \cdot g'(x)$

"Differentiate from outside to inside," and multiply results

1st : $\left(5x+6+\frac{1}{x}\right)^{1/3} \longrightarrow \frac{1}{3}\left(5x+6+\frac{1}{x}\right)^{-2/3}$

2nd : $5x+6+\frac{1}{x} \longrightarrow 5+(-x^{-2})$

inner function (more complex than just x)

$\frac{d}{dx}(x^{-1}) = -x^{-2} = -\frac{1}{x^2}$

$$h'(x) = \boxed{\underbrace{\frac{1}{3}\left(5x+6+\frac{1}{x}\right)^{-2/3}}_{1}\underbrace{\left(5-x^{-2}\right)}_{2}}$$

d. $f(x) = cos^2(x\sin x) = \Big(\underbrace{\cos}_{2}\,\underbrace{(x\sin x)}_{3}\Big)^{\overset{1}{2}}$

1st : $(\cos(x\sin x))^2 \longrightarrow 2(\cos(x\sin x))$

2nd : $\cos(x\sin x) \longrightarrow -\sin(x\sin x)$

3rd : $x\sin x \longrightarrow x\cos x + \sin x$

Product Rule

$$f'(x) = \boxed{\underbrace{[2\cos(x\sin x)]}_{1}\underbrace{[-\sin(x\sin x)]}_{2}\underbrace{[x\cos x + \sin x]}_{3}}$$

3. For what values of a and b is the line $2x + y = b$ tangent to the parabola $y = ax^2$ when $x = -1$?

$y = \boxed{-2}x + b$

(Need $y = mx+b$ form to get slope)

Step 1:

Know $y' = -2$ when $x = -1$:

$$y' = 2ax$$
$$-2 = 2a(-1)$$
$$\boxed{a = 1}$$

Step 2:

$y = ax^2 \longrightarrow y = x^2$

Plug in $x = -1$: $y = (-1)^2 = 1$, so $(-1, 1)$ is a point on the graph (and tangent line)

$$\Longrightarrow 2x + y = b$$
$$2(-1) + 1 = b$$
$$\boxed{b = -1}$$

4. Find dy/dx by implicit differentiation.

$$(y \sin x)' = (y^{2/3} - 3x^2)'$$

Product Rule

$$y \cos x + (\sin x) y' = \frac{2}{3} y^{-1/3} y' - 6x$$

Get y' stuff on one side:

$$(\sin x) y' - \frac{2}{3} y^{-1/3} y' = -6x - y \cos x$$

Factor out y':

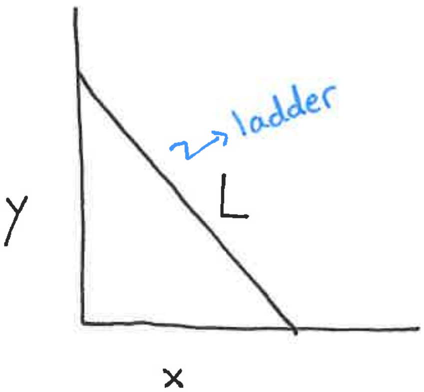
$$y'\left(\sin x - \frac{2}{3} y^{-1/3}\right) = -6x - y \cos x$$

Divide over:

$$y' = \boxed{\frac{-6x - y\cos x}{\sin x - \frac{2}{3} y^{-1/3}}}$$

negative (distance decreases)

5. The top of a ladder slides down a vertical wall at a rate of 0.2 m/s. At the moment when the top of the ladder is 10 m from the floor, the bottom slides away from the wall at a rate of 0.4 m/s. How long is the ladder?



– We are given: $\frac{dy}{dt} = -0.2$, $y = 10$, $\frac{dx}{dt} = 0.4$

– We want: L

Equation (Pythagorean Theorem)

$$x^2 + y^2 = L^2$$

$$\frac{d}{dt}(x^2+y^2) = \frac{d}{dt}(L^2) \quad \text{constant}$$

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$

Plug in: $2x(0.4) + 2(10)(-0.2) = 0$

$$0.8x = 4$$

$$x = 5$$ NOT what we want! We want L!

$$\Longrightarrow x^2 + y^2 = L^2$$

$$5^2 + 10^2 = L^2$$

$$L^2 = 125$$

$$L = \boxed{\sqrt{125}\ m}$$

6. Use a linear approximation (or differentials) to **estimate** the given number. ~~Round to four decimal places.~~

Probably the better method $\uparrow$

$$(8.042)^{1/3}$$

$f(x) = x^{1/3}$

$a =$ "nice number" $= 8$

Linear approx.

Point: $x = 8 \longrightarrow y = f(8) = 8^{1/3} = 2$

The point is $(8, 2)$
$\quad\quad\quad\quad\;\; x_1 \;\; y_1$

Slope: $f'(x) = \frac{1}{3} x^{-2/3}$ $\rightarrow$ flip

$$f'(x) = \frac{1}{3x^{2/3}}$$

$$f'(8) = \frac{1}{3(8^{2/3})} = \frac{1}{3(4)} = \frac{1}{12}$$

The slope is $m = \frac{1}{12}$

$$\Longrightarrow \quad y - 2 = \frac{1}{12}(x-8)$$

$$y = \frac{1}{12}(x-8) + 2$$

Plug in "messy" number $\rightarrow$ $f(8.042) \approx \frac{1}{12}(8.042 - 8) + 2$

$$= \boxed{2.0035}$$

Differentials

$y = x^{1/3}$

$a = 8$

$dx =$ change in $x = 0.042$

$dy = \frac{1}{3}x^{-2/3}\,dx$ (at $a = 8$: $dy = f'(8)\,dx$)

$$f(a + dx) \approx f(a) + dy$$

$$f(8 + 0.042) \approx f(8) + f'(8)\,dx$$

$$= 2 + \frac{1}{12}(0.042)$$

$$= \boxed{2.0035}$$