MATH 110 Summer 2019
Test 2 Practice Problems
(This is NOT a comprehensive review!!!)

Notation/Formulas:

Set Theory

- Cardinality (size) of a set: $n(A)$
- Set operations
  A' (complement) — everything outside of A
  $A \cup B$ (union) — sets A and B listed together
  $A \cap B$ (intersection) — everything in common to both A and B
- Subsets: $A \subseteq B$ — set A is "inside" set B
  ☆ A set with n elements has 2^n subsets. ☆
- The Inclusion-Exclusion Principle (for surveys)
  ☆ $n(A \cup B) = n(A) + n(B) - n(A \cap B)$ ☆
  $n(A \text{ or } B) = n(A) + n(B) - n(A \text{ and } B)$

Logic

$\sim p$	not p
$p \wedge q$	p and q
$p \vee q$	p or q
$p \to q$	if p, then q

- "p if and only if q" is true only when p & q match (i.e., p, q are both T or both F)

- Truth Tables

p	q	$p \vee q$ (or)	$p \wedge q$ (and)	$p \to q$
T	T	T	T	T
T	F	T	F	F
F	T	T	F	T
F	F	F	F	T
		T wins	F wins	The only way for $p \to q$ to be F is if $p = T$, but $q = F$

$\ulcorner\rightarrow$ # of elements

1. Find the cardinality of the following set.

$$\{\underset{2}{\underline{2}}, \underset{2}{\underline{4}}, \underset{2}{\underline{6}}, \underset{2}{\underline{8}}, \underset{2}{\underline{10}}, \dots, \underset{2}{\underline{126}}\}$$

$$\{1, 2, 3, 4, 5, \dots, 63\}$$

$$\Downarrow$$

$$\#\text{ of elements} = \boxed{63}$$

2. Let $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, $A = \{2, 5, 7, 9\}$, $B = \{1, 2, 3\}$, and $C = \{5, 8, 9, 10\}$.
   a. Find $(A' \cup B) \cap C$.
   b. Determine whether the statement is true or false: $C \subseteq B'$

a) $(A' \cup B) \cap C =$ do stuff in ( ) first

$\rightarrow$ everything outside

$(\{2,5,7,9\}' \cup \{1,2,3\}) \cap \{5,8,9,10\} =$

$(\{1,3,4,6,8,10\} \cup \{1,2,3\}) \cap \{5,8,9,10\} =$

$\hookrightarrow$ list together

$\{1,2,3,4,6,8,10\} \cap \{5,8,9,10\} =$

$\hookrightarrow$ everything in common

$\boxed{\{8,10\}}$

b) $C \subseteq B'$

$\{5,8,9,10\} \subseteq \{4,\underline{5},6,7,\underline{8},\underline{9},\underline{10}\}$ ✓ $\boxed{\text{True}}$

(the 1st set is "inside" the 2nd)

3. A Mazda Miata features 10 different upgrade options; you can choose to add any of these when you purchase the car.
   a. How many different versions of Miatas can you buy?
   b. What is the minimum number of upgrade options which must be available if the Mazda dealership advertises that it offers over 5,000 versions of Miatas?

a) We want the number of "subsets" of upgrades:

$$2^n = 2^{10} = \boxed{1{,}024} \text{ versions}$$

b) We want the number of subsets to be at least 5,000:

$2^n > 5{,}000$? Just try numbers!

$2^{10} = \cancel{1{,}024}$

$2^{11} = \cancel{2048}$

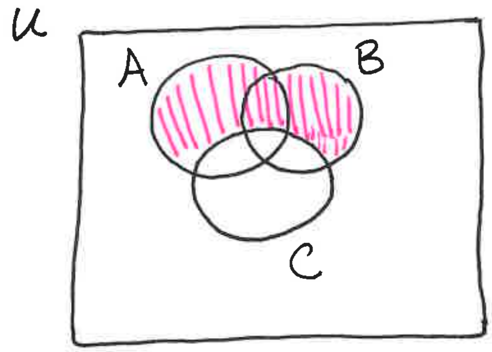
$2^{12} = \cancel{4{,}096}$

$2^{13} = 8{,}192$ ✓ $\longrightarrow \boxed{13}$ upgrades

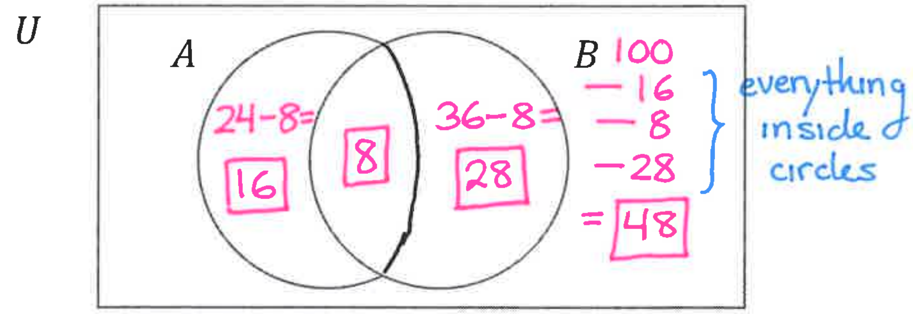
4. Draw a Venn diagram to show the set.

$$(A \cup B) \cap C'$$

↳ the stuff in A or B, but NOT in C



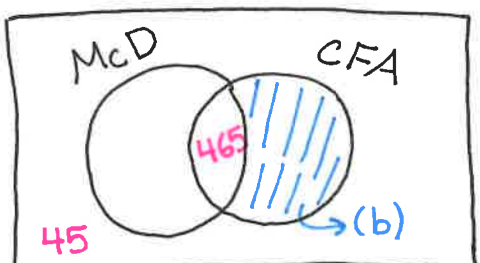
5. Use the given information to find the number of elements in each region.



$n(A) = 24, n(B) = 36, n(A \underset{\text{and}}{\cap} B) = 8, n(U) = 100$

6. In a survey of 1,000 people, it was reported that 670 people liked McDonald's, 750 people liked Chick-fil-A, and 45 people liked neither restaurant.
   a. How many people liked both McDonald's and Chick-fil-A?
   b. How many people liked only Chick-fil-A?

a) Use Inclusion – Exclusion:



n (McD or CFA) = n(McD) + n(CFA) − n(McD and CFA)

total − outside of circles

1000 − 45 = 670 + 750 − n(McD and CFA)

955 = 1,420 − n(McD and CFA)

$\cancel{-}$465 = $\cancel{-}$n (McD and CFA)

⟹ $\boxed{465}$

b) (Chick-fil-A) − overlap = 750 − 465 = $\boxed{285}$

7. Write the sentence in symbolic form. Use the following:
   p: I eat too much.
   q: I order food.
   r: I will feel well.

$(q \quad \wedge \quad p) \quad \rightarrow \quad \sim r$

If I order food and eat too much, then I will not feel well.

↳ group with ( )

$$\boxed{(q \wedge p) \rightarrow \sim r}$$

8. Determine the truth value of the compound statement, given that p is false, q is true, and r is false.

$$(\sim p \vee q) \wedge (p \wedge \sim r)$$

$p = F$ $\quad \sim p = T$
$q = T$ $\quad \sim q = F$
$r = F$ $\quad \sim r = T$

Plug in values: $(T \overset{\text{or}}{\vee} T) \overset{\text{and}}{\wedge} (F \overset{\text{and}}{\wedge} T) =$

$T \wedge F =$

$$\boxed{\text{False}}$$

9. Construct a truth table for the compound statement.

$$(p \rightarrow \sim q) \vee (p \wedge q)$$

Step 1: letters
Step 2: other symbols
Step 3: final column

or, T wins

and, F wins

p	q	$(p$	$\rightarrow$	$\sim q)$	$\vee$	$(p$	$\wedge$	$q)$
T	T	T	F	F	T	T	T	T
T	F	T	T	T	T	T	F	F
F	T	F	T	F	T	F	F	T
F	F	F	T	T	T	F	F	F

10. Determine the truth value of the given statement.

a. If Tuscaloosa is the capital of Alabama, then penguins can fly.

F (Montgomery is!) F $\Rightarrow$ $\boxed{\text{True}}$

If $p = F$, $p \to q$ is automatically true.

b. 3 = 6 if and only if -1 < 1.

F T $\rightarrow$ truth values do not match $\rightarrow$ $\boxed{\text{False}}$