12.2 – Permutations and Combinations

We will continue to learn about methods of counting, using the permutations and combinations formulas.

Motivation

Suppose 4 different colored sheets of paper are arranged in a row. How many different ways are there to order the colors?

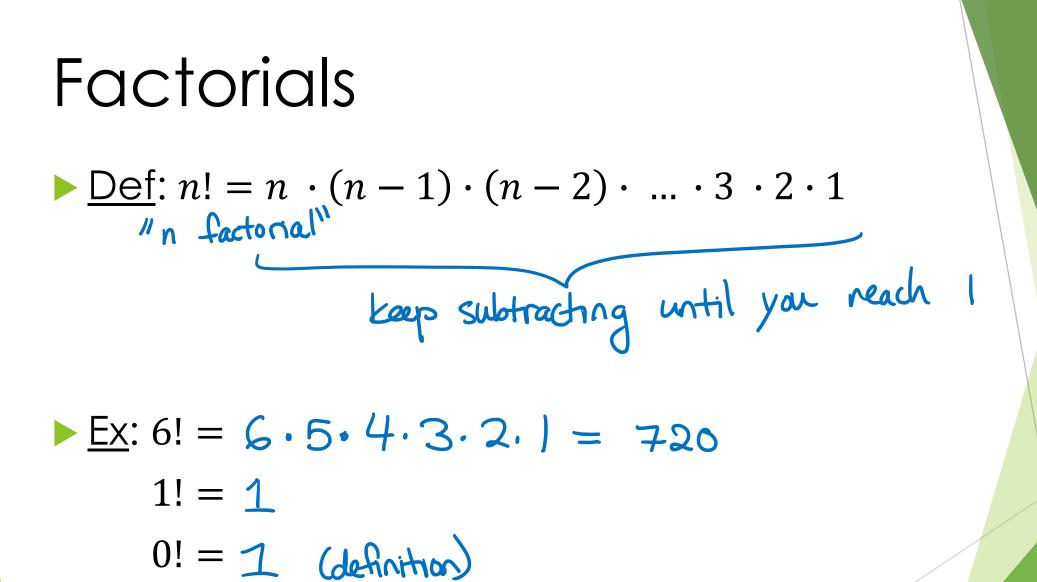
$$\frac{4}{1} \cdot \frac{3}{2nd} \cdot \frac{2}{3rd} \cdot \frac{1}{44h} = 24$$

$$1 = 24$$

$$44h$$

$$44h$$

$$44h$$



Permutations * Order matters *

Def: A permutation is an arrangement of objects in a definite order.

8 runners compete in an Olympic race. In how many ways can the gold, silver, and bronze medals be awarded? P(8,3)

Permutations Formula

The number of permutations of n objects chosen k at a time is

$$P(n,k) = \frac{n!}{(n-k)!} \rightarrow \text{leftovers}$$

Note:
$$P(n, 0) = \frac{n!}{(n-0)!} = \frac{n!}{n!} = \frac{1}{n!}$$

A committee of 16 students must select a president, vice president, secretary, and treasurer. In how many ways can this be accomplished? $16 \cdot 15 \cdot 14 \cdot 13 =$ 680 VP S T $P(16,4) = \frac{16!}{(16-4)!}$ 16.15.14.13.12

Combinations & Order does not matter!

Def: A combination is a collection of objects whose order is not important.

- You put 4 toppings on a pizza and you have 16 toppings to choose from. How many pizzas are possible? —> combinations
- 1. The number of **permutations** is: P(16,4) = 43,6802. This is an overcount! (The order of toppings does not matter.) How many ways can 4 toppings be arranged? $4 \cdot 3 \cdot 2 \cdot 1 = 24$ 3. Divide to correct for the overcount: 43,680

Combinations Formula

The number of combinations of n objects chosen k at a time is $C(n,k) = \frac{\# \text{ of permutations}}{\# \text{ of arrangements}} = \frac{P(n,k)}{k!}$

$$\blacktriangleright \underline{\text{Note}}: C(n, 0) = 1$$

(a) A committee of 6 people is chosen from a group of 12. How many committees are possible? --- combinations $C(12,6) = \frac{P(12,6)}{C'}$ $11 \cdot 10 \cdot 9 \cdot 8 \cdot 7$ •4•<u>3</u>•2 $C(12,6) = \frac{12!}{6! \cdot 6!} =$

(b) A committee of 6 people is chosen from 8 men and 4 women. How many committees are possible that consist of 3 men and 3 women? Multi-stage experiment; use the country principle

$$\frac{c(8,3)}{M} \cdot \frac{c(4,3)}{W} = 56 \cdot 4$$

- (c) 4 cards are randomly chosen from a standard deck of playing cards. How many hands contain exactly 2 queens and 1 king?, 52-4-4-4
 - Counting principle: $c(4,z) \cdot c(4,1) \cdot c(4,1) \quad Q \text{ or } 2Qs \quad 1 \text{ K} \quad 1 \text{ other}$

$$= 6.4.44$$

 $= 1,056$